Modeling and Animation

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Outline

- Modeling
  - Creating the Environment
- Animation
  - Simulating a World in Motion
- Paper Topics
  - Spline Curves

Creating the Environment

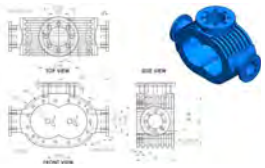
Types of Environment

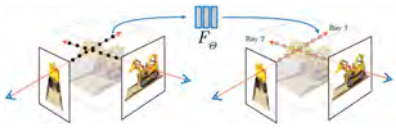
- Indoors
  - Objects
  - Buildings

- Outdoors
  - Urban
  - Landscapes

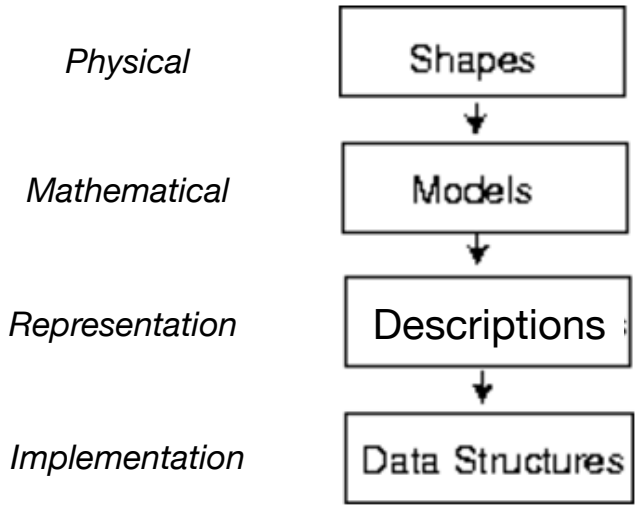
Modeling Techniques

- Traditional Modeling
  - Geometric Modeling [CAD]
    (Man-Made Objects)
  - Procedural Modeling
    (Natural Phenomena)
- Image-Based Modeling
  - 3D Photography
    (Reconstruction of Real Objects)
- AI-Based Modeling
  - Generative Neural Networks
    (Machine Learning of 3D Scenes)





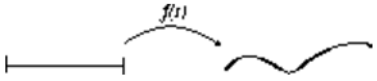
Conceptual Framework



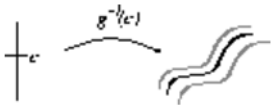
Mathematical Models of Shapes

Functional Specification

- Parametric (*enumeration of points*)
  $(x,y) = f(t)$



- Implicit (*classification of points*)
  $g(x,y) = c$



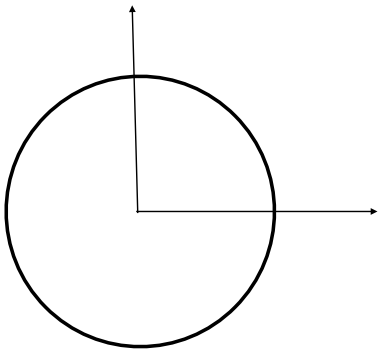
Complementary Descriptions

Example: Unity Circle

- Parametric

$$f(\theta) = [sin(\theta), cos(\theta)]$$

Enumeration of Points



- Implicit

$$x^2 + y^2 = 1$$

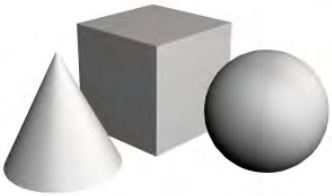
Classification of Points

Traditional Representation Schemes

- Primitive
  - Simple Shapes
- Constructive
  - Combination of Primitives
- Decomposition
  - Stratification in Parts

Primitive Representation

- Shape Library
- Supporting Functions
- (Transformations)



Example: Sphere (*circle of radius r*)

- Parametric Model

$$(x, y) = (r\cos t, r\sin t)$$

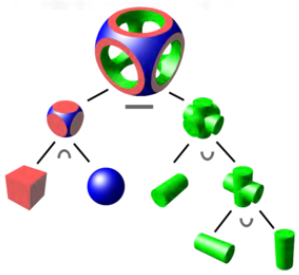
- Implicit Model

$$x^2 + y^2 - r^2 = 0$$

- Translation
- Representation
  - Parameters: (id, r, c)
- Data Structure
  - Associative Array

Constructive Representation

- Half Space Elements
- Combination Operations
  - Point-Set
  - Blending
- Expression Tree



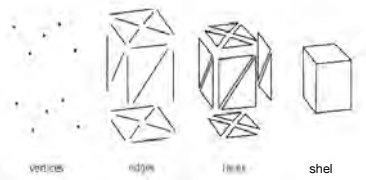
$(c \cup s) - (t \cap (t \cap t))$

Example: Simple CSG

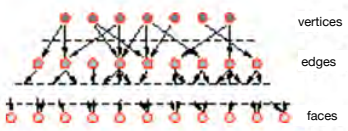
- Model
  - Primitives: Quadrics
  - Operations: union, intersection, difference
- Representation
  - CSG Expression
- Data Structure
  - Binary Tree

Decomposition Representation

- Stratification
  - Vertices
  - Edges
  - Faces
  - Shells



- Topological Graph
  - Layers
  - Links



Example: Polyhedron

- Model
  - Cell Decomposition
  - (Piecewise Linear Geometry)
- Representation
  - Boundary Structure
- Data Structure
  - Polygon List

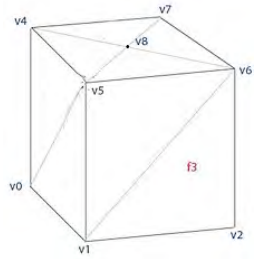
Meshes

Simple Representation

- Polygon: Array of Vertices
- Mesh: List of Polygons

Obs: Other Information at Vertices

- Surface Normals
- Surface Colors
- Texture Coordinates



	Face List
f0	v0 v4 v5
f1	v0 v5 v1
f2	v1 v5 v6
f3	v1 v6 v2
f4	v2 v6 v7
f5	v2 v7 v3
f6	v3 v7 v4
f7	v3 v4 v0
f8	v8 v5 v4
f9	v8 v6 v5
f10	v8 v7 v6
f11	v8 v4 v7
f12	v9 v5 v4
f13	v9 v6 v5
f14	v9 v7 v6
f15	v9 v4 v7

	Vertex List
v0	0,0,0
v1	1,0,0
v2	1,1,0
v3	0,1,0
v4	0,0,1
v5	1,0,1
v6	1,1,1
v7	0,1,1
v8	.5,.5,0
v9	.5,.5,1

A World in Motion

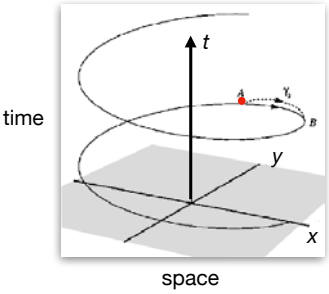
Basics

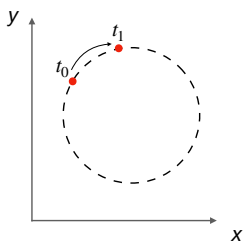
Time-Dependent Phenomena

- Time
  - Clock
- Time Instant
  - Impulse
- Time Interval
  - Continuous

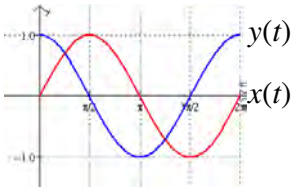
Setting

- Space-Time



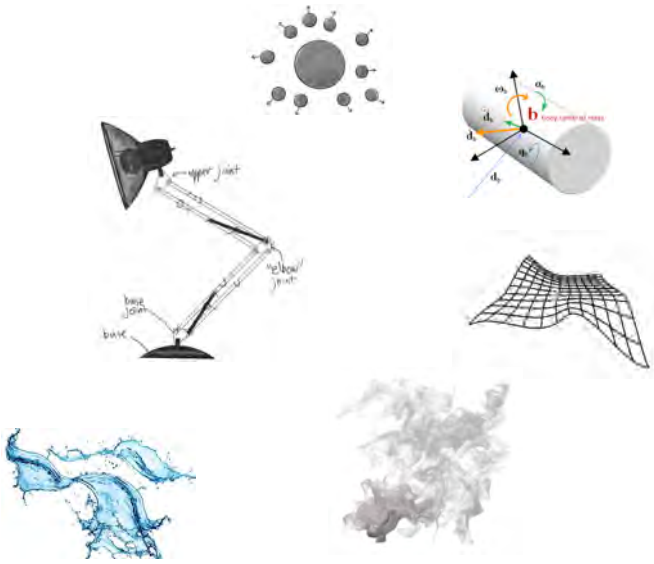


- Time Curves



Bodies

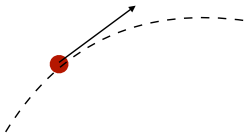
- Particles
- Particle Systems
- Rigid Bodies
- Articulated Structures
- Deformable Objects
- Liquids
- Gases



Mathematical Concepts

- Variation in Time
  - Derivative $\frac{dx}{dt}$
- Motion
  - Differential Equation $\frac{dx}{dt} = P(x(t))$

$$x = \int P(\lambda) d\lambda + C$$

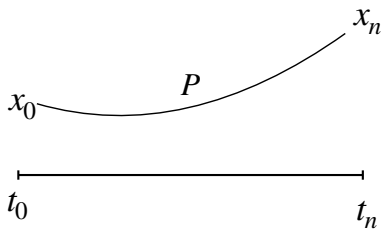


Motion Problems

Find $P(t)$

- Initial-Value Problems
  - *such that*

$$x_0 = P(t_0)$$



- Boundary-Value Problems
  - *such that*

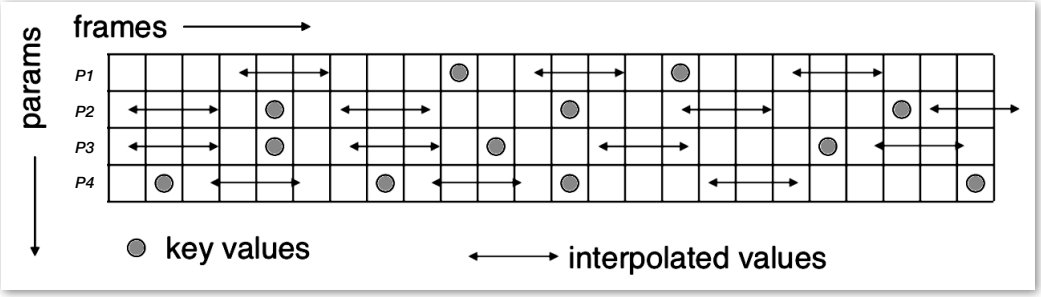
$$x_0 = P(t_0) \quad \textit{and} \quad x_n = P(t_n)$$

Simulation and Animation

- Simulation
  - $P(t)$ is given by the Model of a Process
- Animation
  - $P(t)$ is given by an Algorithm

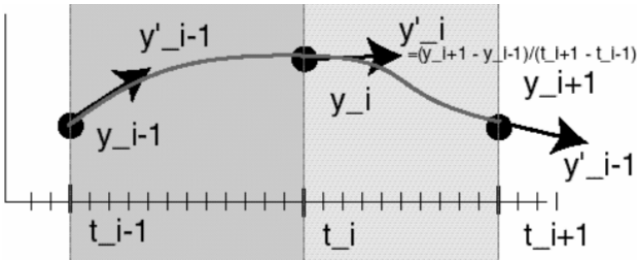
Keyframe Animation

- Parameter Values at Key Frames



Inbetweening

- Interpolation



Polynomial Curves

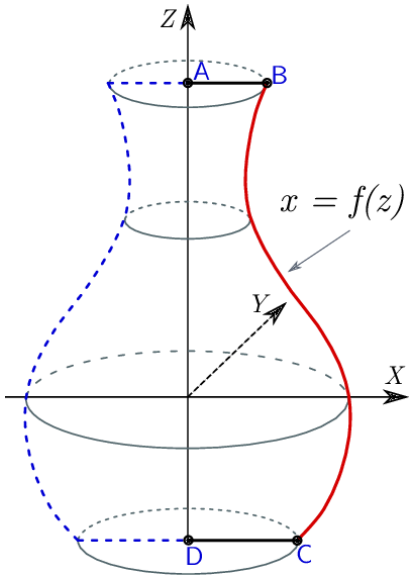
Bezier Curves

- Shape Representation
- Animation Control

✳ Interactive Techniques

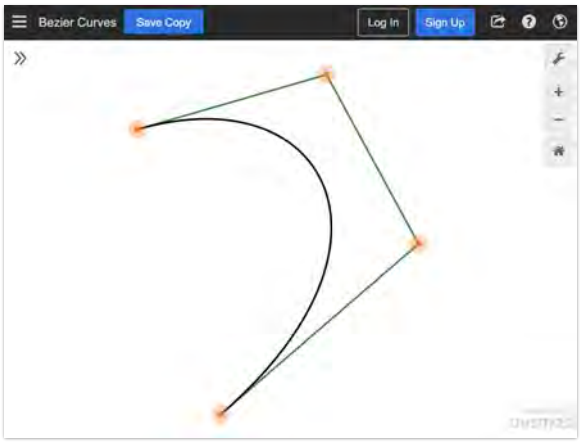
3D Modeling

- Example:
  - Surface of Revolution



Interactive Editing

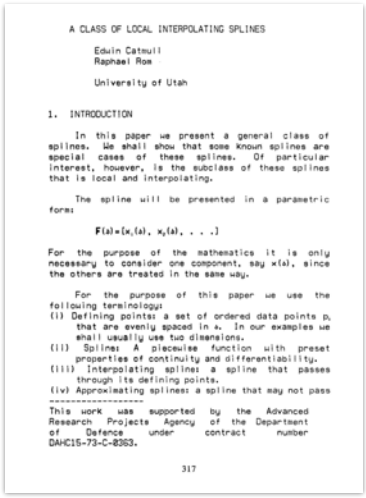
- Demo



A Bit of History

- From Animation to In-Betweening

Animation Pipeline



A CLASS OF LOCAL INTERPOLATING SPLINES

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1. INTRODUCTION

In this paper we present a general class of splines. We shall show that some known splines are special cases of these splines. Of particular interest, however, is the subclass of these splines that is local and interpolating.

The spline will be presented in a parametric form:

$$F(a) = (x_1(a),\ x_2(a),\ \ldots)$$

For the purpose of the mathematics it is only necessary to consider one component, say $x(a)$, since the others are treated in the same way.

For the purpose of this paper we use the following terminology:

(i) Defining points: a set of ordered data points p_i that are evenly spaced in a. In our examples we shall usually use two dimensions.
(ii) Splines: A piecewise function with preset properties of continuity and differentiability.
(iii) Interpolating splines: a spline that passes through its defining points.
(iv) Approximating splines: a spline that may not pass

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Key-Frame In-Betweening