Illumination

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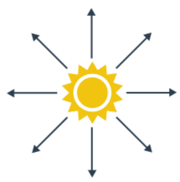
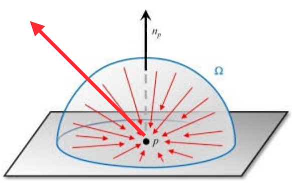
IMPA

Basic Concepts

- Light Sources



- Surfaces / Materials



Illumination Models

- Principle
  - Conservation of Energy
- Electromagnetic Model

  (Local)
  $\Phi_o = F_r \Phi_i + F_t \Phi_i$
  $F_r + F_t = 1$

- Thermodynamic Model

  (Global)
  $\Phi_{tot} = \mathbf{E}_{tot}$
  $\Phi_{out} = \mathbf{E} + \Phi_{in}$

The Rendering Equation

$$\phi(s,\omega) = E(s,\omega) + \int_s k(s,\omega',\omega)\phi(s,\omega')d\omega'$$

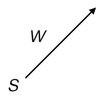
- **Kernel of the integral**
  - Geometry
  - Visibility
  - Reflectivity

Radiant Energy

- **Flux of R.E.**

Phase Space

$$\phi(s, \omega)$$



$$\mathbb{R}^3 \times S^2$$

$$\phi = \frac{dQ}{dt}$$

Radiant energy passing through unity of volume per unity of time

Transport of Energy

- **System in Equilibrium**

$$\frac{\partial \phi}{\partial t} = 0$$

i.e.,

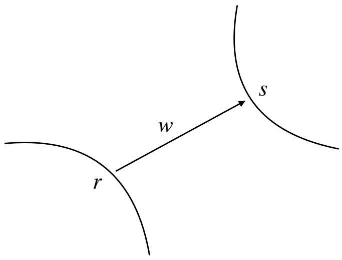
$$\phi = \text{constant}$$

Transport Equation

- Non-participating Medium (*vacuum*)

$$\phi(r,\omega) = \phi(s,\omega)$$

$r, s \in \cup M_i$ and $\omega \in S^2$

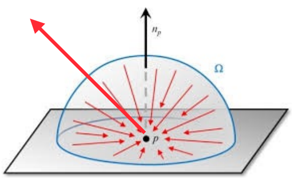


- Visibility Function btw Surfaces

$$\nu(r,\omega) \equiv \inf\{\alpha > 0 : (r - \alpha\omega) \in M\}$$

$$s = r - \nu(r,\omega)\omega$$

Illumination Hemisphere



- Equilibrium of Energy

$$\phi_o - \phi_i = \phi_e - \phi_a$$

outgoing incoming emitted absorbed

Boundary Conditions

- Explicit

$$\phi(s,\omega) = \mathcal{E}(\int,\omega) \qquad s \in \text{Lights}$$

- Implicit

$$\phi(s,\omega) = f_s(\phi(s,\omega')) \qquad \text{scattering function}$$

↑ *out* ↑ *in*

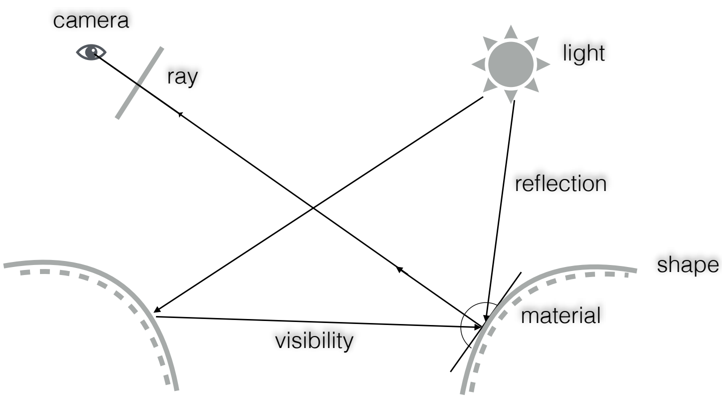
$$\phi(s,\omega) = \int_{\Theta_i} k(s,\omega' \mapsto \omega)\phi(s,\omega')d\omega'$$

OBS: Physical Limitation

$$\int k d\omega \leq 1 \quad \text{and} \quad k \geq 0$$

In Practice

- Rendering Context



Anatomy of the Equation

shape

G

visibility

V

$$L_o(p, w_o) = \boxed{L_e(p, w_o)} + \boxed{\int_\omega} \boxed{f(p, w_o, w_i)} \boxed{L(p, w_i)|\cos\theta|} dw_i$$

camera

ray

light

integration

material

reflection

sampling + reconstruction

Numerical Solution

★ Approximation

- Operator Notation

$$(Kf)(x) = \int k(x, y) f(y) dy$$

Fredholm Eq.
of 1st Kind

- Radiance Equation in operator Form

$$L(r, \omega) = L^e(s, \omega) + (KL)(s, \omega)$$

or

$$L = L^e + KL$$

Method of Substitution

$$L = L^e + K(L^e + KL)$$
$$= L^e + KL^e + K^2 L$$

- repeating

$$= L^e + KL^e + \ldots + K^{n-1}L^e + K^n L$$
$$= \lim_{n\to\infty} \sum K^n L^e$$
$$\approx \sum_{i=0}^{n-1} K^i L^e$$

Intuition: *Bounces of Light*

Approximating the Illumination Integral

$$I = g\mathbf{E} + \mathbf{gMI}$$

$$M = \int k(s, \omega', \omega)$$

- Newman Series

$$(1 - gM)I = g\mathbf{E}$$
$$I = (1 - gM)^{-1} g\mathbf{E}$$
$$I = g\mathbf{E} + \mathbf{gMgE} + \mathbf{g(Mg)^2E} + \ldots$$

Quality of Approximation

- **Error Analysis**
  - Norm of $\| K \| < 1$
- **Residual**

$$e_n = \| \, M_\infty - M_n \, \|$$

- **Physical Interpretation**
  - Direct Lighting (Local)
  - Direct + Indirect Lighting (Global)

Computational Methods

- **Explicit Approximation** (Radiosity / Radiance)
  - Compute L on Surfaces
  - Sample L in Image
  - *Finite Element Methods*
  - Viewer Independent
- **Implicit Sampling** (Ray Tracing)
  - Sample L in Image
  - *Monte Carlo Methods*
  - Viewer Dependent

Direct Lighting

Utah Solution

$$I = g\mathbf{E} + \mathbf{gME_0}$$

- Local Illumination
  (only for light sources)
- No Shadows
- Direct Computation
- No Integration

Indirect Diffuse

$$\phi(s,\omega) = \mathbf{E}(\mathbf{s},\omega) + \int_{\mathbf{S}} \mathbf{k}(\mathbf{s},\omega',\omega)\phi(r,\omega')\mathbf{d}\omega'$$

Kernel of Integral

- Geometry
- Visibility
- Reflectivity

Diffuse

$$k(s,\omega',\omega) = \rho(s)\frac{\cos\theta_a \cos\theta_b}{\pi r_{ab}^2}V_{ab} \quad = \rho(s)F_{a,b}V_{a,b}$$

Visibility

Form Factor

$$\phi(s,w) = E(s,w) + \rho(s)\int F_{a,b}V_{a,b}\phi(r,w')dw'$$

Radiosity

$$\mathbf{E} = (\mathbf{1} - \mathbf{gM})\mathbf{I} = \mathbf{GI}$$

- Global Illumination (diffuse)
- View Independent

Finite Element Solution

$$\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} 1-\rho_1 F_{11} & \ldots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & \ldots & -\rho_2 F_{2n} \\ \vdots & & \\ -\rho_n F_{n1} & \ldots & 1-\rho_n F_{nn} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{pmatrix}$$

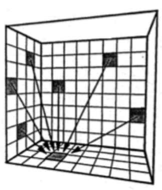
Relaxation Computation

$$I_i^{(k)} = \mathbf{E} + \rho_i \sum_i \mathbf{F_{ij} I_j^{(k-1)}}$$

Radiosity Solution

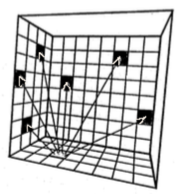
- **Methods**
  - Invert G = $O(n^3)$ *Direct*
  - Iterative = $O(n^2)$ *Gathering*
  - Progressive < $O(n^2)$ *Shooting*

Gathering



Row of F times B
Calculate one row of F and discard

Shooting



Brightness order
Column of F times B

Ray Tracing

$$I = g\mathbf{E} + \mathbf{g}\mathbf{M}_0\mathbf{g}\mathbf{E}_0 + \mathbf{g}(\mathbf{M}_0\mathbf{g})^2\mathbf{E}_0 + \ldots$$

- Global Illumination (specular)
- View Dependent

Stochastic Integration

- Monte Carlo Methods

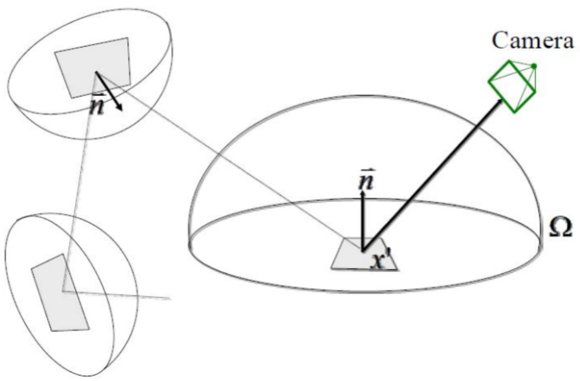
Recursive Computation

- Path Tracing
- Photon Mapping

Path Tracing

- **Random Sampling**

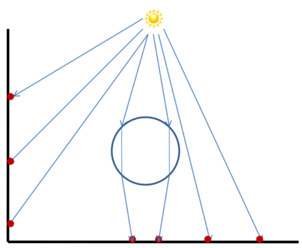
Estimate integral for each pixel by sampling paths from the camera



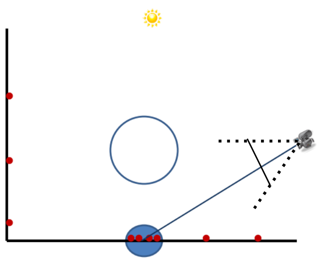
Photon Mapping

- **Two Pass Method**

1. Build Photon Map by tracing random paths from Lights
2. Render Image by tracing random paths from Camera



Map

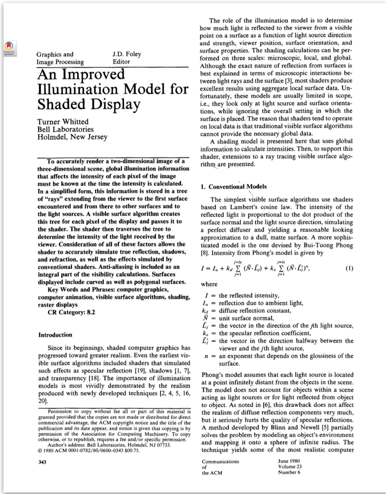


Render

Global Illumination Algorithms

A Bit of History

- From Ideal to Physics Based Illumination

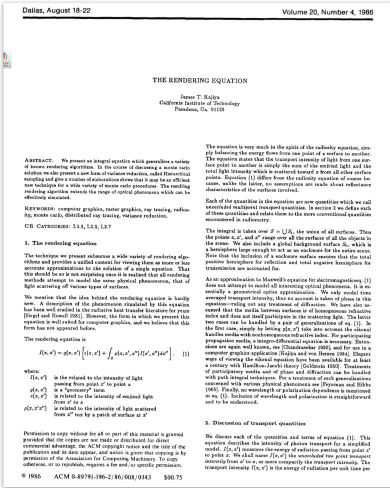


Graphics and Image Processing — J.D. Foley, Editor

An Improved Illumination Model for Shaded Display

Turner Whitted
Bell Laboratories
Holmdel, New Jersey

Ray Tracing



Dallas, August 18-22 — Volume 20, Number 4, 1986

THE RENDERING EQUATION

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Path Tracing

Whitted Ray Tracing

- Recursive Ray Tracing
  - Global Illumination of Ideal Materials (*perfect diffuse & specular)*
- Algorithm:
  1. For each pixel, trace primary ray in direction **V** to the first visible surface.
  2. For each intersection trace secondary rays:
     - Shadow in direction **L** to light sources
     - Reflected in direction **R**
     - Refracted (transmitted) in direction **T**
  3. Calculate shading of pixel based on light attenuation