

# Review of - EWA Splatting: A Novel Framework for Volume Rendering Based on Elliptical Gaussian Kernels

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## 1. Review

### 1.1. Abstract

- List of contributions;

The paper provides a new framework for direct volume rendering using a splatting approach based on elliptical Gaussian kernels. To avoid aliasing artifacts, it introduces the concept of a resampling filter by combining a reconstruction with a low-pass kernel. It provides high-quality images without aliasing artifacts or excessive blurring, even with non-spherical cores.

- Problem addressed;

Aliasing is a fundamental problem of any rendering algorithm, it arises whenever a rendered image or part of it is sampled in one of the pixels.

- Summary of the method;

EWA volume rendering is attractive because it avoids aliasing artifacts in the output image by preventing excessive blurring. Furthermore, it works with arbitrary elliptical Gaussian reconstruction kernels and efficiently supports perspective projection. The authors chose elliptical Gaussians as reconstruction kernels and low-pass filters because they are closed under affine mappings, convolution, and integration of a 3D Gaussian along a coordinate axis results in a 2D Gaussian. Due to their efficiency, the Splatting algorithm belongs to the most popular direct mapping volume rendering techniques. Splatting is attractive due to its efficiency, which derives from using pre-integrated reconstruction cores.

- Describe the method and carry out an analysis justifying each formula used;

Equation (1) is the basis for all splatting algorithms. The

splatting equation (1) represents the output image as a continuous function in screen space. In order to properly sample this function to a discrete output image without aliasing artifacts, it has to be band-limited to match the Nyquist frequency of the discrete image:

$$I_\lambda(\hat{\mathbf{x}}) = \sum_k c_{\lambda k}(\hat{\mathbf{x}}) g_k q_k(\hat{\mathbf{x}}) \prod_{j=0}^{k-1} (1 - g_j q_j(\hat{\mathbf{x}})),$$

where  $q_k(\hat{\mathbf{x}})$  denotes an integrated reconstruction kernel, hence equation (2):

$$q_k(\hat{\mathbf{x}}) = \int_{\mathbb{R}} r_k(\hat{\mathbf{x}}, x_2) dx_2.$$

According to the authors Equation(3):

$$\rho_k(\hat{\mathbf{x}}) = (q_k \otimes h)(\hat{\mathbf{x}})$$

is an ideal resampling filter, combining a footprint function  $q_k$  and a low-pass kernel  $h$ . Then it will be able to analytically integrate the kernels according to Equation (2) and to convolve the footprint function  $q_k$  with a Gaussian low-pass filter  $h$ , yielding an elliptical Gaussian resampling filter  $p_k$ . They also compute the convolution in (3), yielding the EWA volume resampling filter:

$$\begin{aligned}
\rho_k(\hat{\mathbf{x}}) &= (q_k \otimes h)(\hat{\mathbf{x}}) \\
&= \frac{1}{|\mathbf{J}^{-1}||\mathbf{W}^{-1}|} (\mathcal{G}_{\hat{\mathbf{V}}_k} \otimes \mathcal{G}_{\mathbf{V}^h})(\hat{\mathbf{x}} - \hat{\mathbf{x}}_k) \\
&= \frac{1}{|\mathbf{J}^{-1}||\mathbf{W}^{-1}|} \mathcal{G}_{\hat{\mathbf{V}}_k + \mathbf{V}^h}(\hat{\mathbf{x}} - \hat{\mathbf{x}}_k).
\end{aligned}$$

## 1.2. Positive issues

The paper introduced a new footprint function for volume distribution algorithms by integrating an elliptical Gaussian reconstruction kernel and a low-pass filter. The method is based on a new framework for calculating the footprint function, which relies on transforming volume data into the so-called ray space. Due to its flexibility, it can be used to render rectilinear, curvilinear, or unstructured volume datasets. Its main contribution is a new splat primitive that provides high-quality antialiasing and efficiently supports elliptical kernels.

Because its EWA volume resampling filter can handle arbitrary Gaussian reconstruction kernels, it can represent the structure of a volume dataset more accurately by appropriately choosing the shape of the reconstruction kernels.

When mapping the camera to ray space minimizes the volume, the size and shape of the resampling filter are dominated by the low-pass filter. The volume is enlarged and the resampling filter is dominated by the reconstruction kernel. Because the resampling filter unifies a reconstruction kernel and a low-pass filter, it provides a smooth transition between enlargement and minimization.

## 1.3. Negative points

The code has not been optimized for rendering speed. They have not yet investigated whether kernels other than elliptic Gaussians can be used with this structure. The paper does not consider optimizing the code model for rendering speed. Furthermore, it did not investigate how other Gaussian elliptical kernels can be used in this framework.

## 1.4. Evaluation

The paper was approved for publication because it presents an innovative solution. The presentation of ideas was clear and concise. The work can be reproduced based on the information in the paper. All algorithms and mathematical formulas were adequately discussed and the limitations of the work were clearly explained. Therefore, the rating is 5.

Its main contribution is a new splat primitive that provides high-quality antialiasing and efficiently supports elliptical kernels.

## 2. Archeologist

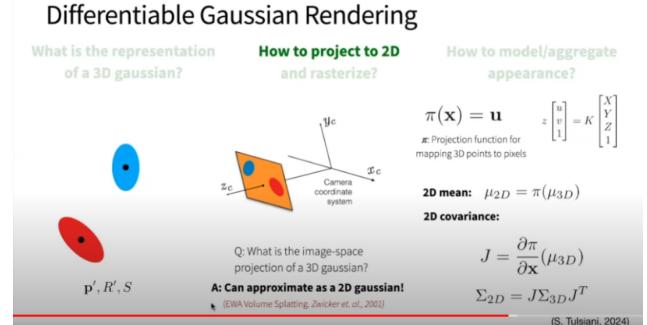
EWA volume splatting [12] is a classic paper, which we are reviewing right now. It got rid from the dust by the 3D Gaussian Splatting (3DGS) [5] success. But there were six former papers before the EWA volume splatting [12] (2001): (1989) [4], (1989) [9], (1990) [10], (1999) [11], Surfels (2000) [7] and (2001) [13]. There are many papers that cite EWA volume splatting [12], but we have identified one following it [8] that improves the algorithm with a hardware approach, the last year's phenomenon 3D Gaussian Splatting (3DGS) [5] that uses EWA as can be seen in [Figure 1](#) and this year's application in medicine [6].

In the "Fundamentals of Texture Mapping and Image Warping" [4] it was cited a former paper reference [Figure 3](#) from the same author, where EWA (Elliptical Weighted Average) appeared for the first time. [Figure 6](#) shows an excerpt from this reference with the key ideas.

[Figure 2](#) shows the paradigm shift from geometry based diagrams to image based paradigm.

[Figure 4](#) and [Figure 5](#) shows the timeline of the publications related to EWA volume splatting [12].

Figure 1. Differentiable Gaussian Rendering.



Source: [S. Tulsiani, 2024]

Figure 2. Properties of real time rendering paradigms.

Table 3.1: Properties of real time rendering paradigms

	Geometry based paradigm	Image based paradigm
Scene	Description in terms of geometry, surface properties, lighting conditions	Description in terms of the plenoptic function
Discretization	Sampling (tesselation) optimized regarding geometric properties	Sampling optimized regarding screen resolution
Representation	Set of primitives (polygons, polygon strips, light sources)	Set of $n$ -dimensional samples
Image synthesis	Conventional rendering	Reconstruction

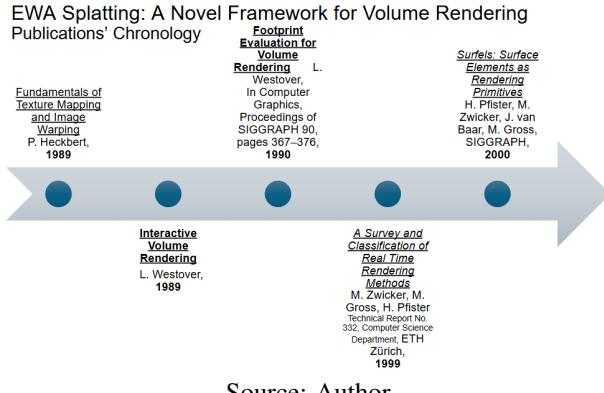
Source:[4]

Figure 3. Former Paper Reference.

[Greene-Heckbert86] Ned Greene, Paul S. Heckbert, "Creating Raster Omnimax Images from Multiple Perspective Views Using The Elliptical Weighted Average Filter", *IEEE Computer Graphics and Applications*, vol. 6, no. 6, June 1986, pp. 21-27.

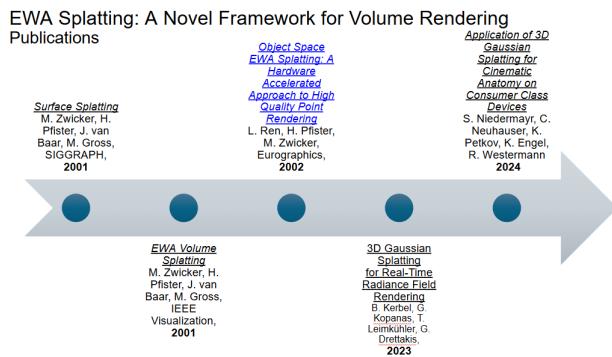
Source:[4]

Figure 4. Chronology 1989-2000.



Source: Author

Figure 5. Chronology 2001-2024.



Source: Author

Figure 6. Excerpt from the reference.

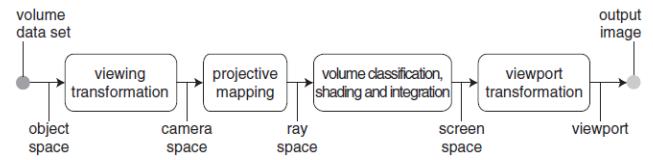
We have seen that the fundamentals of texture mapping and image warping have much in common. Both applications require (1) the description of a mapping between a source image and a destination image, and (2) resampling of the source image to create the destination image according to a mapping function. The first task is a geometric modeling problem, and the latter is a rendering and image processing problem.

To improve the quality of rendering for texture mapping and image warping, we have developed a new theory of ideal image resampling. This theory describes the filter shape needed for perfect antialiasing during the resampling implied by an arbitrary mapping. We have explored one class of filter that conforms nicely to this theory, the elliptical Gaussian, but there are undoubtedly others.

Source:[4]

### 3. Code and experiments

Figure 7. The Forward Mapping Volume Rendering Pipeline.



Source:[12]

From the above block diagram depicted in Figure 7, you can see the forward mapping volume rendering pipeline. There's a mapping from the object (volume 3D) space, to the camera space through a viewing transformation, then from camera space to the ray space through a projective mapping, from ray space to screen space through a volume classification, shading and integration, and finally from screen space to the viewport through a viewport transformation (output image).

The corresponding algorithm is shown Figure 8:

Figure 8. The Corresponding Algorithm.

```

1:for each voxel k {
2:    compute camera coords.u[k];
3:    compute the Jacobian J;
4:    compute the variance matrix V[k];
5:    project u[k] to screen coords.x_hat[k];
6:    setup the resampling filter rho[k];
7:    rasterize rho[k];
8:}

```

Source:[12]

The first line of the code is the iteration for each of the  $k$  voxels. The second one computes the camera coordinates. The third calculates the Jacobian matrix. The fourth transforms the Gaussian kernel from object space to ray space. The fifth projects the kernel to screen space. The sixth corresponds to the resampling filter and the last to the rasterization.

It is available a WebGL implementation of the papers Object Space EWA Surface Splatting: A Hardware Accelerated Approach to High Quality Point Rendering by Ren, Pfister and Zwicker, and High-Quality Point-Based Rendering on Modern GPUs by Botsch and Kobbelt, with a few shortcuts. It also uses the deferred shading for splatting approach described in High-quality surface splatting on today's GPUs by Botsch, Hornung, Zwicker and Kobbelt. The code can be accessed on GitHub [1]

An elliptical weighted average (EWA) surface splatter renderer, implemented in WebGL, which also supports painting on the surfaces. The interactive application can be started from here [2].

The renderer uses an arcball camera which supports mouse or touch input, and downloads datasets via XMLHttpRequest from Dropbox when selected.

Built on top of webgl-util for some WebGL utilities, glMatrix for matrix/vector operations, and FileSaver.js for saving models.

The Dinosaur, Man, Santa and Igua datasets are from Pointshop3D. The Leo dataset is courtesy of Mario Botsch. The Sankt Johann is from the University of Stuttgart. The Warnock Engineering Building is from the State of Utah Wasatch Front LiDAR dataset.

Figure 9. The Dinosaur.

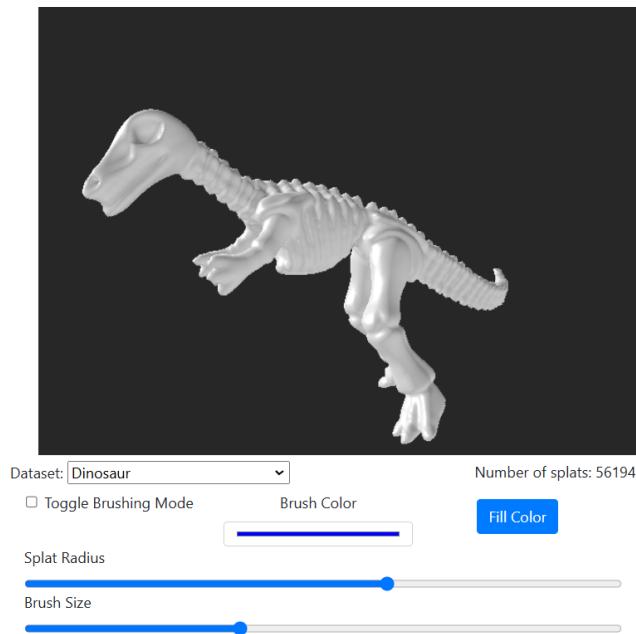


Figure 10. The Dinosaur Sparse.

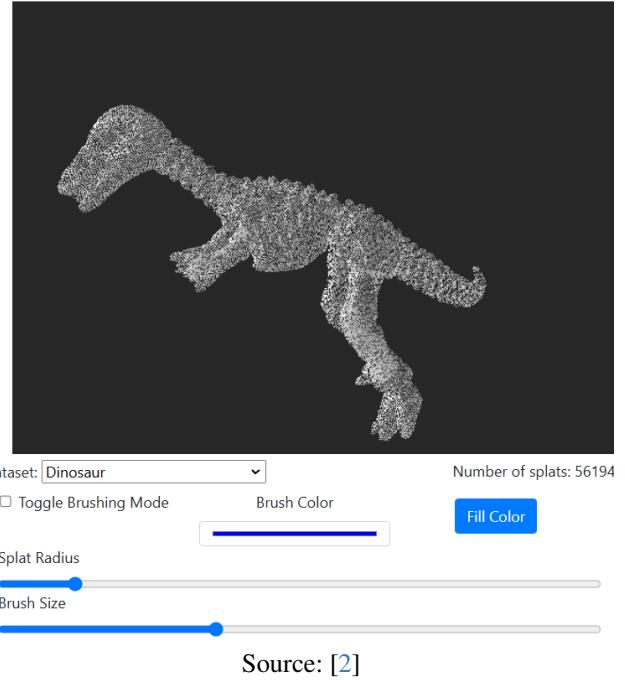


Figure 9, and Figure 10 shows the interactive WebGL EWA Splatter with different values for the Splat Radius configuration. Also in Figure 11 you can see a leopard with different values for the Splat Radius configuration.

## 4. PhD Project

The paper is a milestone in computer vision, being cited to date in papers published at CVPR 2024. Furthermore, its free aliasing quality is superior to that of more recent methodologies and papers. Therefore, we cite its importance for applicability in the medical field that needs clear images with high resolution.

Objectives:

The research project proposal for the doctoral student is the detection of cancer through the conversion of 2d scan images to 3d. The objectives are:

- see how the cancer affected the organ in a three-dimensional way
- predict whether the cancer is likely to spread to other organs
- improve the image quality and view of cancer by re-

searchers and clinicians

- better than 3D mammography, no machine is needed. The image would already come from the exam results with less cost.

- Justifications

EWA volume splatting is a high-quality volume rendering algorithm. It applies the EWA resampling framework to direct volume rendering using a splatting approach, hence avoiding aliasing artifacts. It efficiently handles elliptical reconstruction kernels, which facilitates the visualization of rectilinear and unstructured volume datasets.

- Methodology:

Interactive photorealistic rendering of 3D anatomy is already used in medical education to explain the structure of the human body. It features the use of new view synthesis via compressed 3D Gaussian Splatting (3DGS) to allow students to perform cinematic anatomy on lightweight, mobile devices. The proposed pipeline first finds a set of camera poses that capture all structures potentially seen in the data. High-quality images are then generated and converted into a compact 3DGS representation, consuming 70 MB, even for multi-GB datasets. The implemented method allows the synthesis of new photorealistic visualization in real-time.

According to the National Institutes of Health (NIH), tissues contain a variety of interconnected cells with different functional states and shapes, and this complex organization is impossible to capture in a single plane. Furthermore, tumours have been shown to be highly heterogeneous, requiring large-scale spatial analysis to reliably profile their cellular and structural composition. Volumetric imaging permits the visualization of intact biological samples, revealing dynamic traits of cancer.

The advantage of the method compared to others, such as the use of scanners and 3D mammography machines, is its agility in generating 3D images from 2D images already obtained through conventional exams, in addition to the lower cost.

## 5. Conclusions

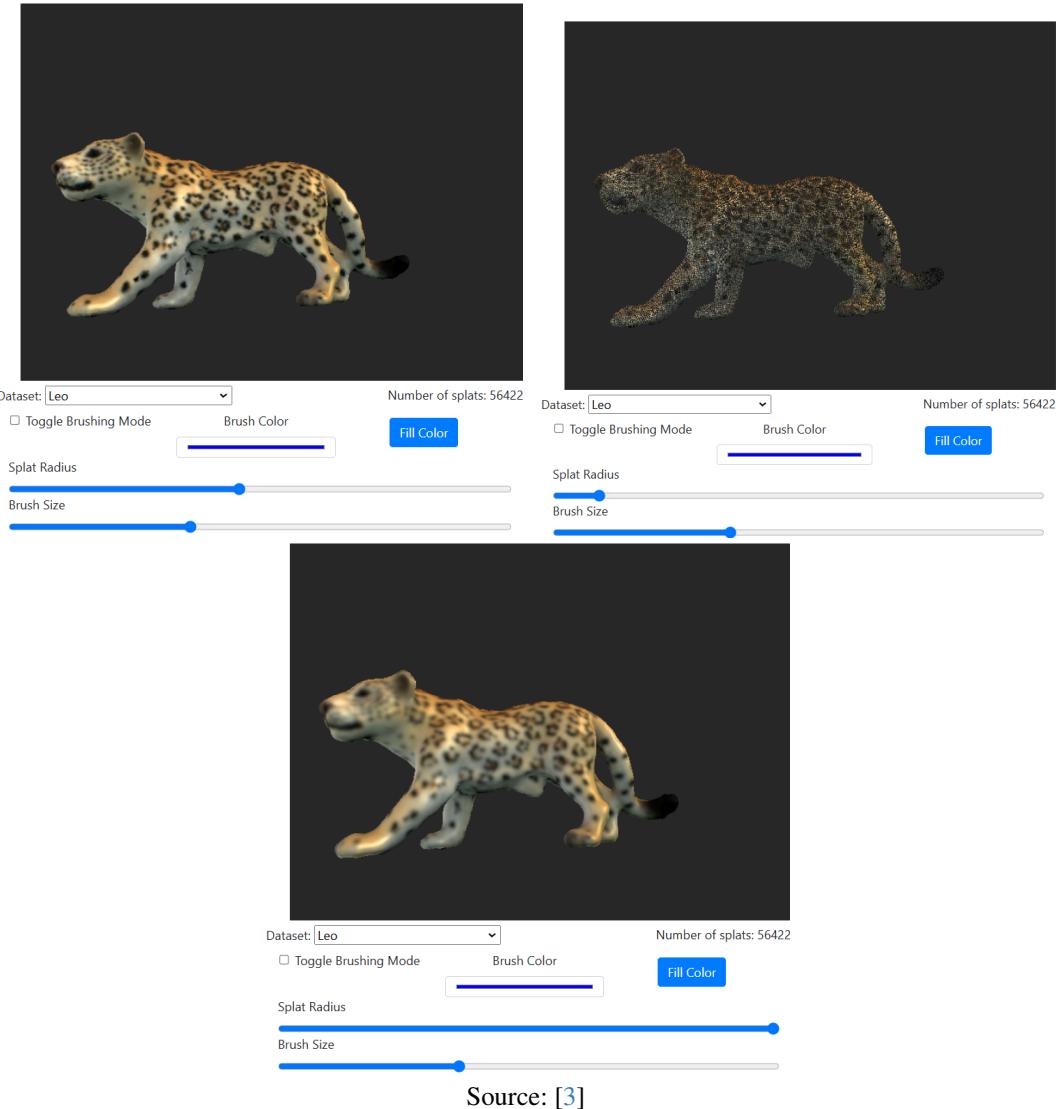
The reviewed paper introduces the EWA (Elliptical Weighted Average) volume resampling filter, a new splat primitive for volume rendering. It combines an elliptical Gaussian reconstruction kernel with a Gaussian low-pass filter to achieve high-quality antialiasing. Using a novel footprint function, their framework efficiently supports el-

liptical kernels and perspective projection, making it suitable for various volume data types. Additionally, they derive a surface reconstruction kernel equivalent to Heckbert's EWA texture filter, making their primitive universal for both surface and volume data reconstruction.

## References

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Figure 11. Leopard with regular splat radius, sparse and full splat radius.



Source: [3]