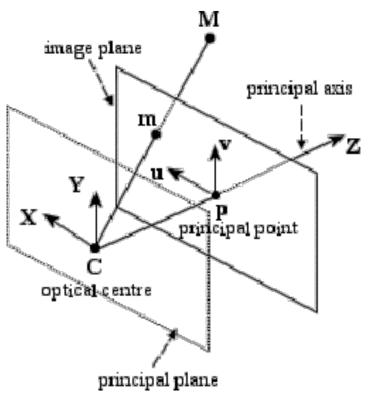
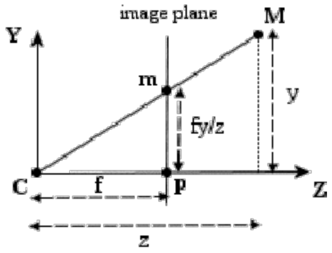
Camera Calibration

Based on Slides from CS558 - U. Washington

Pinhole Camera Model

- Perspective Projection





$v = f\, y/z$

Camera Parameters

Basic:

- focal length
- principal (and nodal) point
- radial distortion
- CCD (image) dimensions
- lens aperture

There is also:

- optical center
- orientation
- digitizer parameters

The Projection Matrix

Matrix Projection: $\mathbf{p} = \begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{MP}$

$\mathbf{M}$ can be decomposed into $\mathbf{t} \rightarrow \mathbf{R} \rightarrow$ project $\rightarrow \mathbf{K}$

$$\mathbf{M} = \underbrace{\begin{bmatrix} fa & c & u_c \\ 0 & f & v_c \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics } (\mathbf{K})} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{orientation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{t}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{position}}$$

Camera Parameters & Projection

Decomposition

- Intrinsics:
  - scale factor ("focal length")
  - aspect ratio
  - principle point
  - *radial distortion*
- Extrinsics
  - optical center
  - camera orientation

How does this relate to projection matrix?

$$\mathbf{p} = \begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{MP}$$

Goal of Calibration

Learn mapping from 3D to 2D

- Can take different forms:

- Projection matrix: $\mathbf{p} = \begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{MP}$

- Camera parameters: $\mathbf{p} = \mathbf{f}(X, Y, Z, \mathbf{K}, \mathbf{R}, \mathbf{t})$

- General mapping $\mathbb{R}^3 \rightarrow \mathbb{R}^2$

Properties of Projection

Preserves

- Lines and conics
- Incidence
- Invariants (cross-ratio)

can show that the only transformations that preserve lines and incidence are the projective transformations

Does not preserve

- Lengths
- Angles
- Parallelism

Calibration Approaches

Possible approaches

- Pattern design
  - planar patterns
  - non-planar grids
- Optimization techniques
  - direct linear regression
  - non-linear optimization
- Cues
  - 3D to 2D
  - vanishing points
  - special camera motions
    - » panorama stitching
    - » circular camera movement

Want

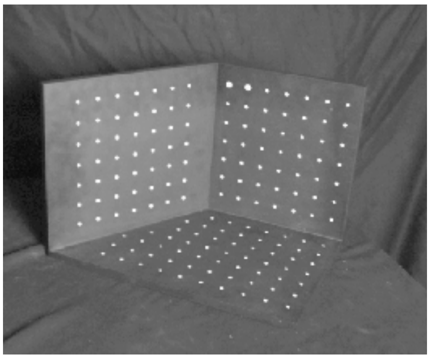
- accuracy
- ease of use

usually a trade-off

A - Estimating the Projection Matrix

Place a known object in the scene

- identify correspondence between image and scene
- compute mapping from scene to image



$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \sim \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & 1 \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Basic Methods for Estimating M

1 - Direct Linear Calibration

2 - Non-Linear Estimation

3 - Statistical Estimation

1 - Direct Linear Calibration

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \sim \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & 1 \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

$$u_i = \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

$$v_i = \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

$$u_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1) = m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}$$
$$v_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1) = m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}$$

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -u_iX_i & -u_iY_i & -u_iZ_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -v_iX_i & -v_iY_i & -v_iZ_i \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \end{bmatrix}$$

- Direct Linear Calibration (cont.)

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & u_iX_1 & u_iY_1 & u_iZ_i \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & v_1X_1 & v_1Y_1 & v_1Z_1 \\ & & & & & & \vdots & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & u_nX_n & u_nY_n & u_nZ_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & v_nX_n & v_nY_n & v_nZ_n \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ \vdots \\ u_n \\ v_n \end{bmatrix}$$

Can solve for m_{ij} by linear least squares

$$minimize \left\| \begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & u_iX_1 & u_iY_1 & u_iZ_i \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & v_1X_1 & v_1Y_1 & v_1Z_1 \\ & & & & & & \vdots & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & u_nX_n & u_nY_n & u_nZ_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & v_nX_n & v_nY_n & v_nZ_n \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \end{bmatrix} - \begin{bmatrix} u_1 \\ v_1 \\ \vdots \\ u_n \\ v_n \end{bmatrix} \right\|$$

What error function are we minimizing?

2 - Nonlinear estimation

Feature measurement equations

$$u_i = \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

$$v_i = \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

Minimize "image-space error"

$$e(\mathrm{M}) = \sum_i [(u_i - \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1})^2 + (v_i - \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1})^2]$$

How to minimize e(**M**)?

- Non-linear regression (least squares),
- Popular choice: Levenberg-Marquardt [Press'92]

3 - Statistical estimation

Feature measurement equations

$$u_i = f(\mathrm{M}, \mathbf{x}_i) + n_i = \hat{u}_i + n_i, \quad n_i \sim N(0, \sigma)$$

$$v_i = g(\mathrm{M}, \mathbf{x}_i) + m_i = \hat{v}_i + m_i, \quad m_i \sim N(0, \sigma)$$

Likelihood of measurements given **M**

$$L = \prod_i p(u_i|\hat{u}_i)p(v_i|\hat{v}_i) = \prod_i e^{-(u_i-\hat{u}_i)^2/\sigma^2} e^{-(v_i-\hat{v}_i)^2/\sigma^2}$$

Negative Log likelihood

$$C(\mathrm{M}) = -\log L = \sum_i (u_i - \hat{u}_i)^2/\sigma_i^2 + (v_i - \hat{v}_i)^2/\sigma_i^2$$

Minimize C wrt. **M**

- gives maximum likelihood estimate (MLE)
- covariance specified by Hessian^{-1} (inverse of second deriv matrix of C)

Camera matrix calibration for M

Advantages:

- very simple to formulate and solve
- can recover **K** [**R** | **t**] from **M** using *RQ* decomposition [Golub & VanLoan 96]

Disadvantages?

- doesn't model radial distortion
- more unknowns than true degrees of freedom (sometimes)
- need a separate camera matrix for each new view

B - Estimating intrinsics / extrinsics Separate

New feature measurement equations

$$\hat{u}_{ij} = f(\mathbf{K}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{x}_i)$$
$$\hat{v}_{ij} = g(\mathbf{K}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{x}_i)$$

i – features
j – images

Use non-linear minimization

- e.g., Levenberg-Marquardt [Press'92]

Standard technique in photogrammetry, vision, graphics

Algorithms

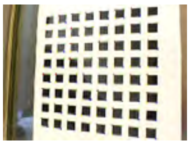
- [Tsai 87] – also estimates κ_1 (freeware @ CMU)
  - http://www.cs.cmu.edu/afs/cs/project/cil/ftp/html/v-source.html
- [Zhang 99] – estimates κ_1, κ_2, easier to use than Tsai
  - code available from Zhang's web site and in Intel's OpenCV
  - http://research.microsoft.com/~zhang/Calib/
  - http://www.intel.com/research/mrl/research/opencv/
  - Matlab version by Jean-Yves Bouget: http://www.vision.caltech.edu/bouguetj/calib_doc/index.html

Calibration from Planes (Tsai and Zhang)

What's the image of a plane under perspective?

- a homography (3x3 projective transformation)

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \sim \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



- preserves lines, incidence, conics

H depends on camera parameters (**A**, **R**, **t**)

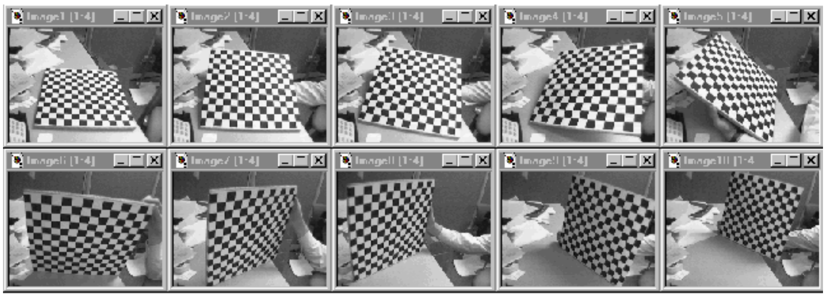
$$H = A\,[r_1\ r_2\ t]$$

where

$$\mathbf{A} = \begin{bmatrix} fa & c & u_c \\ 0 & f & v_c \\ 0 & 0 & 1 \end{bmatrix} \qquad R = [r_1\ r_2\ r_3]$$

Given 3 homographies, can compute A, R, t (Zhang)

Multi-plane calibration (Zhang)



Images courtesy Jean-Yves Bouguet, Intel Corp.

Advantage

- Only requires a plane
- Don't have to know positions/orientations
- Good code available online!
  - Zhengyou Zhang's web site: http://research.microsoft.com/~zhang/Calib/
  - Intel's OpenCV library: http://www.intel.com/research/mrl/research/opencv/
  - Matlab version by Jean-Yves Bouget: http://www.vision.caltech.edu/bouguetj/calib_doc/index.html

Calibration from Planes (Zhang)

1. Compute homography $\mathbf{H}^i$ for 3+ planes
   - Doesn't require knowing 3D
   - Does require mapping between at least 4 points on plane and in image (both expressed in 2D plane coordinates)

2. Solve for **A, R, t** from $\mathbf{H}^1$, $\mathbf{H}^2$, $\mathbf{H}^3$
   - 1plane if only *f* unknown
   - 2 planes if (f, u_c, v_c) unknown
   - 3+ planes for full ***K***

3. Introduce radial distortion model

$$\begin{aligned} \hat{u} &= u + u(\kappa_1 r^2 + \kappa r^4) \\ \hat{v} &= v + v(\kappa_1 r^2 + \kappa r^4) \end{aligned}$$

where

$$r = \sqrt{(u - u_c)^2 + (v - v_c)^2}$$

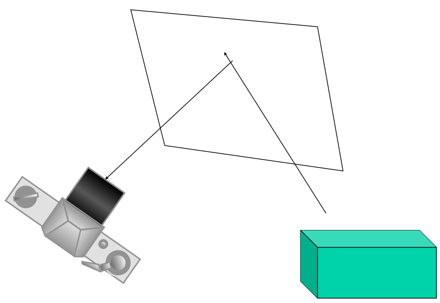
Solve for **A**, **R**, **t**, κ_1, κ_2

- nonlinear optimization (using Levenberg-Marquardt)

Projector Calibration

A projector is the "inverse" of a camera

- has the same parameters, light just flows in reverse
- how to figure out where the projector is?



Basic idea

1. first calibrate the camera wrt. projection screen
2. now we can compute 3D coords of each projected point
3. use standard camera calibration routines to find projector parameters since we known 3D -> projector mapping